

Name: _____

CSUSM Math 160, Fall 2003
Instr: Dr. André Kündgen

THIS IS ONLY A SAMPLE

Final Exam

Show work, **explain your reasoning** and clearly mark your answers. Please ask about anything that is unclear. Use back sides of pages for scratch and overflow work, but clearly indicate which is which!

No books, no notes, no calculators!

1. State the Mean Value Theorem.
2. Integration:
 - a) Give the definition of $\int_a^b f(x) dx$.
 - b) Approximate $\int_1^3 2x + 4 dx$ using a Riemann sum with 4 subintervals and left endpoints.
3. A rectangular region with area 3200 square feet is to be enclosed within a fence. The two sides which run north-south will use fencing material costing \$1 per foot, while the other two sides require fencing material costing \$2 per foot. Find the dimensions of the region which require the least amount of material cost.
4. The length of a rectangle is decreasing at the rate of 1ft/second, but the area remains constant. How many ft/second is the rectangle's width increasing when its length is 10 feet and its width is 5 feet? Show your work!
A) $\frac{1}{2}$ B) 4 C) $\frac{1}{10}$ D) 2 E) 5 F) $\frac{1}{4}$ G) 10 H) $\frac{1}{5}$
5. True or False?
 - a) If $p(x)$ is a polynomial, then $\lim_{x \rightarrow b} p(x) = p(b)$ for every real number b .
 - b) If the line $x = 1$ is a vertical asymptote of $y = f(x)$, then f is not defined at 1.
 - c) $\frac{d}{dx} \ln 10 = \frac{1}{10}$.
 - d) If $f'(x) < 0$ for all x with $1 < x < 6$, then f is decreasing on the interval $(1, 6)$.
 - e) If $f(x)$ is continuous on $[1, 6]$, then $\int_6^1 5f(x) dx = -5 \int_1^6 f(x) dx$.

6. Consider the function $f(x) = \frac{x^2}{x^2-1}$
- Find the domain of $f(x)$.
 - Find all asymptotes of $f(x)$.
 - Determine all critical numbers of $f(x)$.
 - Determine all local/global maxima and minima of f .
 - Use the information above to sketch a graph of f . Label your axis!

7. Compute the derivatives of the following functions:

- $y = \sqrt{\frac{2x-1}{x^2-1}}$.
- $f(x) = \frac{\cos x}{\sin(3x)}$.
- $y = x^{(e^x)}$.

8. Determine the following limits:

- $\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right)$
- $\lim_{x \rightarrow \infty} x \sin\left(\frac{1}{x}\right)$
- $\lim_{x \rightarrow \infty} \frac{\ln x^3}{x^2}$

9. Continuity

- Give the definition of a function being continuous.
- Which of the following functions is continuous (no justification required)? Circle your answers.

$$\text{A) } f(x) = \frac{1}{2} \frac{\ln|x|}{\sqrt{x}} \quad \text{B) } g(x) = x^2 + 1 + e^{-x} \quad \text{C) } h(x) = \begin{cases} \cos(\pi x) & \text{for } x \leq 1 \\ \frac{x^3 - 3x^2 + x - 3}{(x+1)^2} & \text{for } 1 < x \leq 3 \\ \ln(x^2 - 8) & \text{for } x > 3 \end{cases}$$