Math 474, Spring 2004
Assignment 3

due: Thursday, February 12, 2004

1. Combinatorial Arguments, aka Double Counting
   (a) (3.6.15) Prove that $\binom{n}{r} = \binom{n}{n-r}$ using a combinatorial argument, and not Theorem 3.3.1.
   (b) Consider $S = \{n_1 \cdot a_1, n_2 \cdot a_2, \ldots, n_k \cdot a_k\}$ and suppose $n = n_1 + n_2 + \ldots + n_k < \infty$ and that there are p permutations of S. Give a proof of Theorem 3.4.2 different from that in the book by proving combinatorially that $n! = p \cdot n_1!n_2! \cdots n_k!$.
2. (3.6.11) A football team of 11 players is to be selected from a set of 15 players, 5 of whom can only play in the backfield, 8 of whom can only play on the line, and 2 of whom can either play in the backfield or on the line. Assuming that a football team has 7 men on the line and 4 in the backfield, determine the number of football teams possible.
3. (3.6.21) A mathematician works in a building located 9 blocks east and 8 blocks north of her home. Every day she walks 17 blocks to work to stay in shape. (On the grid on page 78 this means she can walk up or to the right along lines only, and she must get from the bottom left to the top right corner.)
   (a) How many different routes can she take?
   (b) Suppose now that the block whose southeastern corner is 4 blocks east and 3 north of her starting point is under water. How many different routes can she take if she wants to avoid walking along this block in order to keep her sneakers clean?
4. Permutations in multisets.
   (a) (3.6.25) Determine the number of 11-permutations of $\{3 \cdot a, 4 \cdot b, 5 \cdot c\}$.
   (b) (3.6.26) Determine the number of 10-permutations of $\{3 \cdot a, 4 \cdot b, 5 \cdot c\}$.
   (c) (3.6.27) Determine the number of 11-permutations of $\{3 \cdot a, 3 \cdot b, 3 \cdot c, 3 \cdot d\}$.
   (d) Generalize (c) by determining the number of $(km-1)$-permutations of $\{m \cdot a_1, m \cdot a_2, m \cdot a_3, \ldots, m \cdot a_k\}$.
5. (3.6.5) Determine the largest power of 10 which divides the following numbers (equivalently, the number of terminal 0's in the ordinary base 10 representation).
   (a) 50!
   (b) 1000!
6. Circular permutations
   (a) (3.6.19) Determine the number of circular permutations of $\{0, 1, \ldots, 9\}$ in which 0 and 9 are not opposite.
   (b) In how many ways can 5 men, 5 women and a dog be placed around a table if no two men are supposed to sit next to each other.
7. (a) (3.6.10) How many sets of 3 numbers each can be formed from $\{1, 2, \ldots, 20\}$ if no two consecutive integers are to be used?
   (b) How many sets of 2 nonconsecutive numbers can be formed from $\{1, 2, \ldots, n\}$?
   (c) How many sets of 3 numbers each can be formed from $\{1, 2, \ldots, n\}$ if no two consecutive integers are to be used?