California State University San Marcos
Math 490
Mathematical Finance
Fall 2008, Course Information

Description: In 1997, Robert Merton and Myron Scholes received the Nobel prize for their work on pricing securities derivatives. An example of a securities derivative is European call option, which is a contract that gives the owner the right, but not the obligation, to purchase a specific security (e.g., a particular stock) at a set time for a set price. By trading such an option, an investor can hedge against the risk associated with market volatility. For example, suppose that an investor believes that the price of a certain stock will be high at some time in the future. Rather than purchasing the stock, the investor could instead purchase a call option. Then, the investor can decide whether or not to exercise the option based on the actual stock price.

Pricing securities derivatives emerged as an important concern in the late 1960's and early 1970's when a relaxation in the regulations allowed insurance companies and banks to invest in the derivatives market. At about that time, two economists, Fisher Black and Myron Scholes, developed a mathematically based pricing strategy, which Robert Merton later simplified and expanded. This work had a profound impact on the trading practices in financial markets worldwide. Sadly, Black passed away prior to 1997, and therefore was ineligible to be a co-Nobel prize recipient.

In the course, we will examine a simplified version of this body of work. In particular, we will analyze the multi-period Cox-Ross-Rubenstien (CRR) model and the finite market model using some of the key principles and methodologies that Black, Scholes, and Merton developed to construct their derivatives pricing theory. Despite the fact that these models are very simple, the analysis will illustrate certain essential aspects of their work such as dynamic hedging and the risk neutral probability measure. We will use the analysis of the multi-period CCR model to dervive some of the Black-Scholes pricing formulas. Finally, if time permits, we will also uses the risk neutral proability to solve certain portfolio optimization problems.

Objectives: The aim of the course is to introduce students to stochastic modeling via the the study of models for financial markets.

Student Learning Outcomes (SLOs):
According to the 2008-10 CSUSM Catalogue **Mathematics B.S. students should**:

1. Master the core concepts in algebra and analysis.
2. Give clear and organized written and verbal explanations of mathematical ideas.
3. Develop and write mathematical proofs.
4. Solve mathematical problems independently.

5. Use appropriate technology to solve mathematical problems.
6. Understand and apply algorithms to solve problems.
7. Model and analyze real world problems by reformulating these problems in a mathematical context.
8. Recognize the interdependency of different areas of mathematics, the connections between mathematics and other disciplines, and the historical context for the development of mathematical ideas.

The course will address these SLOs as follows. Proof techniques will be heavily based in linear algebra, and in particular will make use of the structure and properties of vector spaces. Through homework and exams student will develop and demonstrate their proficiency with providing written explanations of mathematical ideas. Homework and exams will present students with opportunities to develop and write proofs, and also to solve mathematical problems independently. Calculators will be used in this course as appropriate. The exposure to algorithms will be limited to finding replicating strategies, so this SLO is reinforced here, but not a main theme of the course. The main question that the content of the course seeks to answer comes from financial markets and therefore modeling and analyzing real world problems by reformulating these problems in a mathematical context will be demonstrated in spades. This question has a rich and interesting development beginning in the early 1900's, which will be incorporated into the course content. Finally, the connection of mathematics with economics and also the connections between finite probability and linear algebra will be made apparent throughout the course.

According to the 2008-10 CSUSM Catalogue **Mathematics M.S. students should**:

1. Achieve mastery of the foundations of one or more advanced areas of mathematics.
2. Write extended passages of mathematical prose following modern conventions of precision and clarity.
3. Explain advanced mathematics orally following modern conventions of precision and clarity.
4. Produce mathematical proofs in advanced areas of mathematics.
5. Understand, and critique for accuracy, complex mathematical proofs.
6. Understand, produce, and critique mathematical models and algorithms appropriate to their fields of specialty, utilizing appropriate software where necessary.
7. Understand, appreciate, and explain the motivation and culture of their field(s) of specialty. This includes the major historical developments of the field, and the connections between the field other areas of mathematics and science.

8. Master the techniques, proofs and applications of differential and integral calculus, and apply the methods of calculus in a variety situations, such as analyzing numerical methods, ordinary differential equations, partial differential equations, measure theory, complex analysis, applicable analysis, and differential geometry.

This course is appropriate for beginning graduate students. As such they will be acquiring some of the foundations necessary for fully attaining the 8 graduate SLO's. Graduate students will study in detail the Master of Science Thesis of Serena Mercado, which will be of great value to them as they seek out and produce their own thesis projects.

Location/Time of Course Meetings: Arts 220, Mondays & Wednesday, 5:30pm-6:45pm.

Instructor: Dr. Puha; Sci2-325; 750-4201; apuha@csusm.edu;
`http://www.csusm.edu/puha`.

Office Hours: Mondays 7pm in Arts 220;
Mondays & Wednesdays 12pm-12:45pm in Sci2-325.

Virtual Office Hours: Questions sent via email will be responded to during office hours, if time permits. Students physically present have priority.

Email Policies: To avoid having your message routed into the instructor's spam folder, please send messages from your CSUSM student email account. You are responsible for reading all messages sent to your student email address.

Course Web Page: `http://courses.csusm.edu/math490ap/`.

Required Textbooks:

1. Ruth J. Williams. *Introduction to the Mathematics of Finance*. American Mathematical Society, 2006. Available online at `http://www.ams.org/bookstore`.

2. Serena Mercado. *The Mathematics of Pricing Contingent Claims in Incomplete Markets Using Discrete Stochastic Models*. Cal State San Marocs MS Thesis, 2008. Available from the instructor at a cost of $16.00.

Additional References:

1. Stanely R. Pliska. *Introduction to Mathematical Finance: Discrete Time Models*. Blackwell, 1997.

2. J. Hull, Options, *Futures and other Derivative Securities*, Prentice Hall, Fourth Edition, 2000.

3. Sheldon M. Ross and Erol A. Pekoz. *A Second Course in Probability*. `http://www.ProbabilityBookstore.com`, Boston, MA, 2007.

4. S. Ross, *An Introduction to Mathematical Finance*, Options and other topics, Cambridge University Press, 1999.

5. J. Stampfli and V. Goodman, *The Mathematics of Finance: Modeling and Hedging*, Brooks/Cole, Pacific Grove, CA, 2001.

6. P. Wilmott et al., *The Mathematics of Financial Derivatives*, Cambridge University Press, 1995.

Prerequisites: Must have passed each of the following courses with a grade of C or better.

1. Discrete Mathematics (Math 370), or fluency in reading, understanding, and doing mathematical proofs.

2. Introduction to Mathematical Probability and Statistics (Math 440), or sufficient mastery of the course content. In particular, students need to know about sample spaces, random variables, expected value, and conditional expectation. To download a free introductory probability book, visit `http://www.dartmouth.edu/~chance/teaching_aids/books_articles/probability_book/book.pdf`.

3. Linear Algebra (Math 374), or sufficiently strong understanding of solving high dimensional systems linear equations. Of particular importance will be to have a strong understanding of linear independence, rank, basis, span, and dot product.

Missed Lectures: It is your responsibility to get the lecture notes from one of your fellow classmates in the event that you are able to attend a lecture.

Homework: Homework assignments will be assigned via the course webpage and due at the BEGINNING of lecture on the due date. Clear, complete, fully justified solutions are required for full credit. Presentation also counts. Papers must be stapled and answers must be legible and well organized. NO LATE HOMEWORK will be accepted. Questions are welcome in office hours. Students are expected to make an ernest effort to solve the problem and to clarify their questions before seeking help. Some assignments will have a different point value than others, so instead of dropping the lowest score or lowest percentage, 50 of the homework points will be treated as bonus points.

Turning Homework in Early: If for some reason you do not plan to come to lecture on the day that homework is due and want to get credit, you can place it in the instructor's mailbox on the third floor of Science Hall 2 AT LEAST 30 minutes prior to the time of the lecture meeting.

Midterm Exams: Two midterm exams will be given. Each counts for 300 points. To prepare, students should review the homework problems and the lecture notes. NO MAKEUP EXAMS. Plan accordingly.

Midterm Dates:
Exam One: Monday, October 13.
Exam Two: Wednesday, November 26.

Final Exam: The exam is comprehensive and counts for 600 points of the grade. Please bring a self addressed, stamped postcard if you would like your grade mailed to you.

Final Exam Date: Monday, December 8, 4pm-6pm.

Grades: Your grade will be calculated from the best of the following options:

Homework	Exam One	Exam Two	Final Exam	Total
20%	20%	20%	40%	100%
20%	0%	20%	60%	100%
20%	20%	0%	60%	100%

Academic Honesty: Academic dishonesty will NOT be tolerated. Violations will be reported to the Dean of Students

Academic Honesty and Homework: The instructor recognizes that students benefit from discussing approaches to the assigned homework with their classmates. This practice is encouraged. However, it is expected that each student will make an independent attempt to solve any given homework problem and prepare questions before discussing it with his/her classmates. It is also expected that each student will submit an independent final write up of his/her solution. To achieve this, the final write up should be completed without referring to a book or notes and without asking others for assistance. The student should continue to study and ask questions until a final independent write up can be achieved.

Academic Honesty and Learning Resources: Students are only permitted to use resources specifically intended for student use in this class. In particular, there are NO circumstances under which students are permitted to use a resource intended for the instructor or made available by another instructor. If you find a resource that you would like to use and aren't sure whether it is permitted, simply ask the instructor in advance.

Cell Phones: TURN THEM OFF during class.

Approved Calculators: The TI-30X IIB and TI-30X IIS are the ONLY calculators that are approved for use in this class. Please bring your calculator to each class meeting. NO other electronic devices are to be out on your desk or turned on during class. This includes ALL other calculators, cell phones, and laptops. Such calculators are often available at local electronics stores. They can also be purchased online at `http://education.ti.com/educationportal/sites/US/productCategory/us_scientific.html`.