# Math 540, Fall 2004
# Assignment 5

due: Thursday, December 2, 2004

All $(v,k,\lambda)$-designs in this assignment are nontrivial, that is they satisfy $3 \leq k < v$. Furthermore, a design is called *simple* if no block in it is repeated.

1. Let $\mathcal{B}$ be an $(n^2,n,1)$-design, that is an affine plane, and let $B$ be any block in $\mathcal{B}$.
   (a) Show that for any $x \notin B$, there is a unique block that contains $x$, but is disjoint from $B$.
   (b) Show that there are exactly $n-1$ blocks that are disjoint from $B$.
   (c) Show that the blocks in (b) form a parallel class, and use this to argue that every affine plane is resolvable.
   (d) Prove that an affine plane of order $n$ exists if and only if a projective plane of order $n$ exists.

2. Derivations. Let $\mathcal{B}$ be a $(v,k,\lambda)$-design with set of varieties $V$.
   (a) Show that the *complement* $\overline{\mathcal{B}} = \{V - B : B \in \mathcal{B}\}$ is a design, and determine its parameters.
   (b) Show that if $\mathcal{B}$ is symmetric and $D \in \mathcal{B}$, then the *residual design* $\{B - D : B \in \mathcal{B}\}$ is indeed a design, and determine its parameters.
   (c) Show that there is a (13,9,6)-design.

3. (a) Show that there is a unique STS(7) up to isomorphism. (Hint: w.l.o.g. the first two blocks are 124,156.)
   (b) Show that there is a unique STS(9) up to isomorphism.
   (c) Let $\mathcal{B}$ be a STS($v$). Define a matrix $A = (a_{ij})$ by $a_{ii} = i$ and $a_{ij} = k$ if $\{i,j,k\} \in \mathcal{B}$. Show that $A$ is a symmetric Latin square.

4. (a) Show that every $(6,3,2)$-design is simple.
   (b) Show that there is a unique $(6,3,2)$-design up to isomorphism.
   (c) Show that there is a unique $(11,5,2)$-design up to isomorphism.
   (d) Show that there is a unique $(11,6,3)$-design up to isomorphism.

5. Determine all parameters $(v,k,\lambda)$ for which there is a $(v,k,\lambda)$-design with $r \leq 5$. Are all these designs unique up to isomorphism?

6. For each of the following parameters find a difference set with these parameters, or explain why none exists.
   (a) $(v,k,\lambda) = (15,7,3)$.
   (b) $(v,k,\lambda) = (19,9,4)$.
   (c) $(v,k,\lambda) = (56,11,2)$.
   (d) $(v,k,\lambda) = (57,8,1)$.
   (e) $(v,k,\lambda) = (79,13,2)$.