Take Home Final Exam

Due: Thursday May 11, 2006, 4:30PM in my office

General Instructions:

- *You are expected to work on the final by yourself – in other words, treat it as an in-class exam that you are allowed to take at home.* **Discussions of any kind regarding this final are not permitted**, *except that you may consult with me if you have questions.*
- *You may consult other books or material, but if you do and you use something specific on a question, your solution must state clearly the source of your help and must not be plagiarized.*
- *Do not wait to start the exam! It will require some time and your solutions are expected to be complete and thorough (since you have more time than on an in-class exam). You will not be given additional time beyond the due date, so start early and not the night before!*

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1. (8.1.57) Let G be a connected 3-regular plane graph in which every vertex is incident to one face of length 4, one face of length 6, and one face of length 8. Without drawing G, determine the number of faces of G. (12 pts)
2. (8.1.85) Let T_1, T_2, T_3 be the three directed trees formed by the internal edges in a Schnyder labeling. Let G be the directed graph obtained by deleting the external edges and reversing the edges of T_1. Prove that G has no cyclically oriented cycles. (12 pts)
3. For $n \geq 4$ let M_n be the graph obtained from the cycle C_n by adding chords between vertices that are opposite (if n is even) or nearly opposite (if n is odd). The graph M_n is 3-regular if n is even, and 4-regular if n is odd.
   (a) (8.3.17) Determine the crossing number $\text{cr}(M_n)$ of this graph. (8 pts)
   (b) Determine $\chi(M_n)$. (10 pts)
   (c) Determine the acyclic coloring number of M_n when n is even. (10 pts)
4. (8.4.6) For every natural number $n \geq 9$ such that n is not prime, or twice a prime, construct a 6-regular simple toroidal graph with n vertices. Determine the weak coloring number of the embedding you obtain for as many choices of n as you can, and for all other values give as tight an upper and lower bound as you can. (10+8 pts)
5. (9.3.8) Let e be any edge in a bridgeless graph G. Prove that if $G - e$ has a positive k-flow, then G has a positive $(k + 1)$-flow. (12 pts)
6. (3.4.15) *List-coloring:* Given a list assignment L to the vertices of G, let $L(X) = \cup_{v \in X} L(v)$ for all $X \subseteq V(G)$.
   (a) Prove that if the subgraph $G[X]$ of G which is induced by $X \subset V(G)$ is colorable from these lists whenever $|L(X)| < |X|$, then G is L-colorable. (10 pts)
   (b) Prove that if G is L-colorable for every k-list assignment L such that $|L(V(G))| < |V(G)|$, then G is k-choosable. (8 pts)
   (c) Use (b) to prove that the complete multipartite graph with k parts and 2 vertices in each part, $K_{2,\ldots,2}$ has list-chromatic number k. (10 pts)
7. From your classmates' talks, pick one (other than your own) that you enjoyed the most and in your own words, describe the key results or the proofs that intrigued you. You are not asked to regurgitate the proofs, but to give a sense of why you consider the results significant or interesting and how it ties in with the material we covered. (10 pts)