

## Assignment 4

due: Thursday, April 6, 2006

1. (8.1.15) Determine all choices of  $n$  so that there exists a (simple) 4-regular planar graph on  $n$  vertices.
2. (8.2.38) Prove that the vertices of a (plane) triangulation  $G$  can be partitioned into two sets inducing forests if and only if the dual graph  $G^*$  has a spanning cycle. (A graph that has such a cycle containing all of its vertices is called *Hamiltonian*.)
3.
  - (a) (8.2.25) Prove that a (plane) triangulation is 3-colorable if and only if it is *Eulerian* (i.e. it is connected and every vertex has even degree).
  - (b) (8.2.26) Prove that a planar graph is 3-colorable if and only if it is a subgraph of an Eulerian triangulation.
4.
  - (a) (8.1.67) Use Kuratowski's Theorem to prove that  $G$  is outerplanar if and only if  $G$  does not have a subgraph which is a subdivision of  $K_{2,3}$  or  $K_4$ . (Hint: Trying to mimic the proof of Kuratowski's for outerplanar graphs is much harder than applying Kuratowski's Theorem directly to a modification of  $G$ .)
  - (b) State and prove an outerplanar version of Wagner's Theorem, that is characterize outerplanar graphs in terms of forbidden minors.
  - (c) Use the Four Color Theorem to prove that every outerplanar graph is 3-colorable.
5. (8.2.23) Prove the *Art Gallery Theorem*: If an art gallery is laid out as a simple polygon on  $n$  sides in the plane (see picture on page 331 of Chapter 8), then it is possible to place  $\lfloor n/3 \rfloor$  guards such that every point in the interior of the gallery is visible to some guard. For every  $n \geq 3$  construct a polygon on  $n$  sides that can be guarded by no fewer than  $\lfloor n/3 \rfloor$  guards. (Hint: use the fact that every outerplanar graph is 3-colorable.)
6. Choosability of outerplanar graphs:
  - (a) Determine all 2-connected outerplanar graphs that are 2-choosable.
  - (b) Use your answer to part (a) to characterize the choice number of an outerplanar graph in terms of its blocks.